Complex Interactions

in

Learning and Problem Solving

by

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CHAPTER I

Theoretical Overview

Those experienced and observant teachers of academic subjects which have problem solving as a major goal may have had the opportunity to observe first hand the diversity of approaches used by their students in learning and problem solving. Students' learning and problem solving styles and strategies are generally self-taught, and perhaps not necessarily optimal. Pask(1976) points out that generally speaking not only are learning strategies of students mis-matched with the teaching strategies of teachers, but also students themselves may opt for learning strategies for which they lack competence. In this regard, therefore, it is the complex interactions of student capabilities, student learning and problem solving strategies, and instructional strategies that may be the determining factors of success in formal education.

Cronbach and Snow (1977) discuss in detail the elusive aptitude by treatment interactions (ATI) and how ATI's once found tend to be unstable. An example of an ATI would be where students of a given aptitude (e.g high inteligence) learn better under one teaching strategy, or treatment, than a second. However, students low in that same aptitude learn better under the second teaching strategy than the first. An experiment, in this regard, that has a null finding either (1) has shown that an ATI does not exist, or (2) has shown that the experiment lacked sufficient sensitivity to detect an effect (Kerlinger and Pedhazur, 1973). ATI's were uncovered in many studies reviewed by Cronbach and Snow. ATI's were found as well in an experiment designed by the present writer (McCormick & Mouw, Note 1). Along with the ATI's found was also an aptitude by aptitude interaction (AAI). The interaction of two aptitudes may be exemplified by the following: Some problem solving may be a function of the ability to transform mental images which is in turn dependent upon the ability to form and hold mental images. So, the regression slope of problem solving on the ability to transform images is dependent on the ability to form the image initially. This sort of AAI was found in another investigation conducted by the present writer (McCormick, Note 2). Adding a learning strategy or an orientation to problem solving makes the possible complex interactions nearly as complex as learning might expected to be. In this same investigation a significant complex aptitude by aptitude by orientation triple interaction (AAOI) was revealed. This AAOI was the multiple interaction of spatial abilities, span of short term memory and an orienting response to statistics coursework material. Two classes of graduate level statistics students formed the sample. The teaching strategy was uniform across the sample studied. Hence, with teaching strategy added as a treatment variable, one may presume to expect a complex aptitude by aptitude by orientation by treatment quadruple interaction, significantly accounting for outcome in coursework.

It is this complex issue of higher-order interactions that this study intends to address. The aptitudes or abilities of particular interest are spatial abilities (McGee, 1979; Smith, 1964), and M-space (Pascual-Leone, 1970; Case 1972, 1975, 1977). The orienting response of interest can be attributed primarily to Piaget's (1977) concept of "nonbalance." This orienting response has several components which can be expressed as dichotomies: (1)work vs. play (lexical), (2) product, or goal, vs. process attitude (Dewey, 1933), (3) accomodation vs. assimilation (Piaget, 1977), (4) substantive vs. metaphorical modes (Samples, 1975), (5) differentiation vs. integration (Piaget, 1977). The teaching strategies of interest were best described by Pask (1976) as "holistic" vs. "serialistic" or "comprehension" vs. "operation" teaching. The criteria of interest are ecologically valid (Neisser, 1976) measures of problem solving skills. These will be scores derived from the final exam of a graduate level statistics course for the social sciences. The goal of this study is not to "prove" that outcome is better for a given treatment type. Rather the goal is to find, if at all possible, a better matching of abilities, learning strategies and teaching strategies. This can be done by focussing attention on the complex multiple interactions that are possible.

Variables of Interest

Along with the variables of interest described above, two other variables, sex and years of college mathematics experience, have previously been considered in studies of mathematics achievement.

Math Experience

Years of mathematics experience is an obvious predictor of success in a course which is mathematically oriented (deWolf, 1977). However, in terms of understanding (as opposed to prediction) underlying processes, the abilities and general orientation themselves should predict the number of years of college math. Years of college math may be used in a multiple regression model in order to increase the sensitivity of the significance test (Kerlinger and Pedhazur, 1973) so long as there is no significant correlation between years of math and the other independent variables. An investigation, referred to above (McCormick, Note 2), in which years of college math was included in a regression model revealed no significant contribution over and above ability, affect, and orientation variables.

Sex

Much has been written about sex differences in math achievement (Eisenburg & McGinty, 1977; Fennema, 1974, 1975; Guay & McDaniel, 1977; Jacklin & Maccoby, 1972; McDaniel & Guay, 1976), which indicates that with sex differences as a main effect males tend to do better than females in math achievement. However, the literature indicates that when differences in spatial abilities are controlled for, sex differences generally tend to disappear (McGee, 1979). Data collected as part of research conducted by McCormick & Mouw (Note 1) indicates that sex difference uniquely accounts for variance in problem solving scores over and above spatial abilities. For their sample there was a non-significant correlation between sex and spatial abilities. Eisenburg & McGinty (1977) have data to indicate that males and females tend to select courses appropriate to their abilities, and that females in business statistics, for example, tend to have generally higher spatial scores than males. So, apparently sex differences may fluctuate depending on content domain of the criterion. Again, just as with years of college math experience, McCormick (Note 2) found that the effects of sex differences disappeared in the presence of ability, affect, and orientation variables.

<u>Spatial Abilities</u>

Smith (1964) concluded from his comprehensive review of spatial abilities that they involve a person's ability to treat stimulus material in a holistic fashion.

That is, it is the ability to at once deal with the total relationships of all the parts of a configuration. Different from this assesment of Smith's is Piaget and Inhelder's (1971) comprehensive study of the development of mental imagery. They analyze "mental imagery" or the "spatial sphere" of cognition into six components: static, kinetic and transformational, reproductive or anticipatory imagery. This cognitive imagery depends heavily on the level of cognitive development of the person. The range is from reproductive, static imagery at the lowest level of imagery development to anticipatory, transformational imagery at the higher end. So, in Piagetian terms, a spatial test having these levels of imagery is a measure of operational level, while in Smith's terms it is a measure of one's ability to engage in holistic thought.

M-Space

Pascual-Leone (1970) defines M-space as the number of "figurative" and/or "operative" schemes that can be held in short term memory at one time. Mental problem solving or learning may involve what Piaget (1977) calls the "reciprocal assimilation of schemes" and the "reciprocal accomodation" of schemes. These processes bring together two or more conceptual schemes into a single emergent scheme. Miller (1956) would call this process "chunking". A short term memory (metaphorically a work bench) large enough to allow those schemes to undergo a reciprocal assimilation or chunking, is large enough to construct complex conceptual schemes. Cognitively the complexity of concepts in one's conceptual repertoire will be determined by one's M-space, since from an individual's point of view concepts are constructed piece-wise in short term memory.

Balance

The construct of balance is more complex and hence difficult to explain. However, Dewey (1933) believed that a balance of product and process attitudes towards a task determines how effectively it is accomplished. He states, "To be playful and serious at the same time is possible, and it defines the ideal mental condition." Play at its extreme is "foolishness", while work at its extreme is a "drudgery". Taking a playful attitude is essentially giving the mind freedom to make unusual connections, while work attitude merely means to keep one's thoughts in a general progression towards a goal.

Piaget indicates that play is an essential part of problem solving, that it is essentially means devoid of ends (process removed from production), and that play is "an assimilation of reality into the self". (Phillips, 1975) Assimilation is the "incorporation of an outside element (object, event, and so forth) into the subject's sensorimotor or conceptual scheme." (Piaget, 1977)

The complement of this process is accomodation, which is a creation of or change in ones cognitive structure to allow assimilation to follow. Accomodation may also result from an assimilation. A person in "balance" with his/her environment has assimilation and accomodation in balance. One form of balance is between the transformation of the element being assimilated and the adjustments or accomodations of the system or subsystem into which it is being assimilated. A second form of balance occurs between subsystems of the subject which define the subject's conceptual schemes. In the process of reciprocal assimilation of schemes there must as well be a reciprocal accomodation of schemes. "Non-balance" (Piaget, 1977) occurs when one scheme absorbs the other at a faster rate. A third form of balance is between the rates of differentiation and integration of conceptual schemes. He suggests that this third form of balance is very much related to the other two forms. The reason he suggests for the "initial nonbalance" is the "systematic primacy of the positive characteristics of observations..." That is, people tend to see only similarities between an external system and the conceptual scheme into which they attempt to assimilate that external system. The nonbalance is a "disturbance" which "is merely an obstacle holding an assimilation in check (whether it derived from a fact contradicting a judgement or from a situation which prevents achieving a goal)" (Piaget, 1977). That is, further assimilation into the chosen scheme is obstructed. How one responds to the obstruction or non-balance is classified by Piaget into three types of behaviors. In type α behaviors one ignores contradicting characteristics or else distorts them to fit his/her chosen scheme. In type β behaviors, one modifies both the contradicting characteristics and the chosen scheme so that they become assimilable. In type γ behaviors, one anticipates possible contradictions and hence avoids any non-balancing effect.

Samples (1968, 1976) has focused mainly on the natural problem solving behaviors of children. He observed that children given a problem to be solved start off in a "metaphorical mode". In this mode children say that this is like such-and-such, or this is a such-and-such. Once they obtain a useful metaphor or analogy they switch into a "substantive mode", where they direct their efforts toward the problem solution as a goal. That is, they work out the ideas they gained in play.

The natural pattern of problem solving appears to be a balance of playful and work attitudes, but starting primarily in a playful attitude and moving into a work attitude. Samples observed that children tended to jump back-and-forth between the two modes, but always starting in the metaphorical or playful mode.

Teaching Strategies

Pask (1976) describes two categories of students, "Holists" and "Serialists", and suggests that teaching strategies can be tailored to these two groups. A third category of students he calls "versatile" would do well with either strategy. Pask describes the two as follows: "Evidence suggests that the holist is assimilating information from many topics in order to learn the 'aim' topic, while the serialist moves on to another topic only when he is completely certain about the one he is currently studying."

Teaching to the serialist would require an instructor to teach a subtopic thoroughly before going on to the next. At each step the teacher may refer back only to the one previous step and show how it relates to the present step. Procedures and operations would be taught; and each step of a procedure would be demonstrated before going on to the next.

Teaching to the holist would require covering in a general, non-specific way all the subtopics. After this, the interrelatedness of the subtopics to what has already been learned must be made. Not until the global picture has been described (and how the procedures and operations fit into that global picture) can the teacher go on to demonstrate the steps of the procedure.

tend to assimilate several subtopics together into any scheme that pops into STM. During a holistic teaching strategy he/she would not need to coordinate several subtopics at once, since they would fit directly into his/her scheme of assimilation. One should expect that the correlation between statistics scores (with holistic teaching strategy) should be positive for all values of M-space (Figure 1c).

Persons unbalanced in favor of work (accomodation, imitation, etc.) under the influence of a holistic teaching strategy would generally respond to statistics coursework as if it were a memory task. Those who are capable of dealing with course material in a holistic fashion (high spatial) again would do well if their M-space is larger; since several more subtopics can be "chunked" together at a time than with a smaller M-space. Those who are mismatched with a holistic teaching strategy by having low spatial ability, either attempt to use a learning strategy for which they lack ability or else fall back on another strategy for which they are able. Those with higher M-space would have a false sense of understanding during a holistically oriented lecture because he/she is able to keep in mind many ideas and relationships that are under consideration. However since they lack the holistic processing ability (spatial ability), the coordinated subtopics undergo relatively little processing. Those with lower M-space generally will not understand anything going on in lecture, and will resort to his/her prolific notes to learn the material later. Five out of five people in McCormick's (Note 2) data who were in this category (low spatial, low M-space and product oriented) indicated that they take "a lot of notes" in order to learn the material later. (See Figure 1a).

Persons in a class with a serialist strategy of teaching need only an M-space of two in order to coordinate the present subtopic with the last subtopic in the serial of topics. A student desiring to see how a subtopic relates to all others (not just the preceding one) may not have his efforts rewarded by his/her serialist teacher. That is, a holistically capable student will do generally more poorly than a serialist in this situation. (Pask, 1976) This means that if one tends to have process and product in balance, there will be a negative correlation between statistics coursework and spatial ability. And the degree of this correlation will be unaffected by the M-space of the person.

However, as balance "tilts" in favor of process (assimilation or play), the holistically capable student may make the interrelationships among subtopics on his/her own for their own sake. It is because of this it is expected that there will be a positive correlation relatively little processing. Those with lower M-space generally will not understand anything going on in lecture, and will resort to his/her prolific notes to learn the material later. Five out of five people in McCormick's (Note 2) data who were in this category (low spatial, low M-space and product oriented) indicated that they take "a lot of notes" in order to learn the material later. (See Figure 1a).

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When the balance tilts in favor of the work orientation it is expected that the holistically capable student would view the coursework as a drudgery. Such a student would attempt to accomodate his/her learning style capabilities to the teacher's style with less success than his low spatial counterpart. So, again a negative correlation (irrespective of M-space) between statistics and spatial score.

<u>Research Hypotheses</u>

I. (Cross validation) Given McCormick's (Note 2) regression equation

Statistics = 0.4 *Spatial+0.4*Mspace+0.4*Balance
+0.5*(1- Balance)*Spatial*Mspace,

where Statistics = predicted statistics course points (inclusive of experimental outcome score described below)

Spatial = spatial abilities score (standardized)

Mspace = M-space value (standardized)

Balance = "ork vs. Play orientation (McCormick, Note 2) (Standardized),

there will be a significant correlation between statistics scores predicted by this equation and actual statistics course scores.

II (Experimental)

a) The degree to which the difference between a "holistic" vs. "serialistic" teaching strategy (in graduate level statistics for the social sciences) is related to outcome of this teaching depends on student's spatial ability. (Spatial by teaching strategy interaction hypothesis evolved from Pask, 1976).

b) The degree to which a spatial ability by M-space interaction is related to teaching outcome depends on the level of nonbalance (nonbalance being the degree to which assimilation was blocked and consequently accomodation was required). (Spatial by M-space by balance interaction hypothesis, McCormick, Note 2).

c) The degree to which a spatial ability by M-space by balance interaction is related to teaching outcome depends on teaching strategy. (Spatial by Mspace by balance by treatment interaction hypothesis based on logical extropolation of data).

Importance of this Study

This study will be one of many studies that have particularly investigated interactions of abilities and treatment . While this study is primarily of theoretical interests rather than practical interest to this writer, it would hopefully provide information leading to better matching of student abilities, strategies for learning and strategies for teaching. In an egalitarian society and especially in schools supported by public tax monies, selection of students for curriculae in general and courses in particular should be on the bases of matching ability to teaching strategy. The tradition is to randomly assign students without regard to such matching. This is analogous perhaps to randomly assigning hospital patients to treatment groups (e.g. a patient with a broken leg being sent to maternity). We cannot expect to be called "educators" if we expect our students to accomodate their abilities and learning strategies to our teaching styles. It is felt that once the complex interactions involved in learning are understood, then perhaps we can teach "anyone anything" he/she would like to learn. Much more work than this study alone will provide such understanding.

Limitations of this Study

The measure of general "balance" is based on a self-report questionnaire. This reduces reliability considerably. The measure of balance may perhaps follow the same history as imagery, which started off with self-report questionaires, which in turn later had little relation to measures of cognitive imagery (Forisha, 1975). However, at the present time there is no ecologically valid measure of this construct other than obtaining self-reports. In self-report measures, the best that can be done is to limit loaded questions that imply cultural norms or "shoulds". Brown (1976) suggests that self-report measures tend to be valid in situations where faking responses has no reward. He points out that the unstable nature of some self-report inventories may be due more to the phenomenon being measured than the instrument attempting the measurement. He says that in most cases "the appropriate method of establishing validity is construct validity." This takes years of work.

CHAPTER II

Review of the Relevant Literature

Balance as a Main Effect

While it is the intent of this study to concentrate on the interaction of variables, it is worth while to look at each variable as a main effect. This may be, in a Hegelian sense, like trying to understand an emergent property by centering attention on the "thesis" and "antithesis". However, perhaps our understanding of the emergent property of learning (which statistically reveals itself as interaction of main effects) may in turn be the emergent property of our understanding the components of the interactions.

Piaget's (1977) work is essentially the essence of 50 years of labor and literally hundreds of experiments with dozens of collaborators. He uses metaphors that he admits to be primarily biological in nature, but does occasionally branch out into the physical sciences for metaphors and analogies to make a point. His two postulates in his "theory of equilibration" uses the biological metaphors of assimilation and accomodation. The two postulates are as follows:

"First postulate: Any scheme of assimilation tends to feed itself, that is, to incorporate outside elements compatible with its nature into itself." This means that a person by virtue of being alive tends to integrate into his action or conceptual schemes those elements that are consistent with the nature of that scheme. A child may see a cow and call it a dog, simply because he/she integrated that which is similar between a cow and a dog into the already established conceptual scheme for a dog.

"Second postulate: The entire scheme of assimilation must alter as it accomodates to the elements it assimilates; that is, it modifies itself in relation to the particularities of events but does not lose its continuity...nor its earlier powers of assimilation". This is essentially a "postulate" that assimilation and accomodation must remain in balance "in order for the accomodation to succeed"... The "scheme of assimilation" means specifically the action or conceptual scheme into which an element is assimilated. As in the example above: when the child saw the cow, the scheme of assimilation was the concept dog. If gone uncorrected, the child's conceptual scheme would have accomodated to the "particularities of events" by including in his/her conceptual scheme for dog the possibility that a dog may have horns and as well go "mooo!" as well as bark. Usually a child is thrown into a state of nonbalance by a parent who calls the child an idiot and states knowingly that it is not a dog but a cow.

Piaget suggests that there are three kinds of equilibration, or balancing. The first occurs during a subject-object interaction. "There is the equilibration between the assimilation of schemes of action and the accommodation of these to the objects." The balance then has to be maintained between bringing an object into an action scheme (e.g. a stick or rock being brought into the action scheme grasping) and the modification of the action scheme to the object (e.g. opening one's hand wide for a large object). Non-balance occurs in this sense when an object cannot be assimilated into an action scheme (e.g. one cannot grasp a smooth wall).

The second form of equilibration occurs during the interactions of the "subsystems" of the subject, that is, the interaction of action schemes or conceptual schemes. Two or more conceptual or action schemes equilibriate, or come into balance, through reciprocal assimilation and accomodation. For example, the three action schemes of paddling one's feet, moving one's arms and moving one's head side to side while breathing can all undergo a balancing of reciprocal assimilation and accomodation of each other to form the emergent scheme of swimming. The three schemes accomodate to each other by becoming a coordinated motion. They have "assimilated" each other by coming together as a single scheme of swimming. On a conceptual level, the two schemes "F-distribution" and "within and between group estimates of population variance" undergo a balancing of reciprocal assimilation and accomodation to become "analysis of variance F-test of significance." Non-balance is caused here by the fact that "the subsystems are generally constructed at different speeds." e.g. one may understand one concept better than a second, and so therefore missing components of the second make an emerging concept incomplete.

The third form of equilibration is that between integration and differentiation of schemes ("subsystems")." On the one hand, to differentiate a totality, T, in subsystems, S, means not only to confirm what each of these possesses but also to exclude, or deny, the characteristics each subsystem does not have. On the other hand, to form (to integrate) a total system, T, means to free positively the characteristics common to all the S, but this also means to distinguish - this time negatively - the common features of special characteristics not belonging to T." An example of integrating into a totality while balancing the differentiation of subsystems is the following: light, sound, water waves and earthquake motion may all be integrated (assimilated) into a totality "Wave Phenomena". The common characteristics of wavelength, frequency, speed, polarizability, etc. are the characteristics all in common to the totality. But at the same time one differentiates special characteristics not in common with the totality, e.g. media of propagation of the waves. An example of nonbalance of this form is the child who has a strong sense of belonging (having been integrated) in his/her environment, but yet lacks his/her individuality. The child regains balance so-to-speak by asserting his/her individuality (differentiation) and making his/her own contrary decisions. (Damen, 1979).

Piaget (1977) identifies three types of behaviors through which people proceed in order to maintain balance.

Type α behaviors: "When a new characteristic is incompatible with a previous discernment, the subject, though perceiving it, will neglect or will pretend to consider it, but distort it in order to adjust it to the scheme retained for the discernment." Such behaviors have earned such phrases in the lexicon as "pre-conceived notions", or "prejudice", etc. Here the person maintains equilibrium or balance either by refusing to interact freely with his/her environment, or else the person assimilates the new characteristic _after_ it has been transformed. This sort of transformation can help assimilate the new information only so long as it is "a small disturbance close to the balancing point." If it is a large disturbance it will probably be ignored.

Type β behaviors: "Modifying the system by 'equilibrium displacement' so that the unexpected fact is assimilable. The description will thus be improved; the classification will be recast to coordinate the new category with the others..." Piaget's 'equilibrium displacement' is like the lexical "changing ones mind." If an African child sees a European for the first time changes his ideas to include human beings having light skin, pointed noses and straight hair, then the child has modified his system by 'equilibrium displacement'. The perceived European is assimilable into a human being scheme.

Type γ behaviors: the anticipation of the "possible variations which, as forseeable and deducible, lose their disturbance characteristics and establish themselves in the potential transformations of the system." Those who have the imagination or ability to forecast outcomes tend not to be thrown into nonbalance when a new characteristic is hit upon. The scientists at Los Alamos in 1945 generally were not "disturbed" when their atomic bomb went off. They, through their operations upon their constructs of reality, anticipated what would happen when two pieces of Uranium-235 of particular size were pushed together. This they anticipated though it had never been done before. We all are not "disturbed" when we let lose a ball and it falls to the floor. We anticipate as much, but would be rather upset (or even excited) if it "refused" to succomb to gravity.

Both Phillips (1975) and Ginsburg and Opper (1979) bring Piaget's theoretical abstractions "down to earth", by putting Piagetian Theory in more concrete, lexical, and perhaps measurable terms. Phillips describes the development of "play" as a process of differentiation of "means" separate from "ends." Phillip states: "The separation of means from ends has far reaching implications. When the two are completely separated and the ends drops out (the means become an end in itself), we have <u>play</u>; when they are differentiated but continually related, we have <u>problem-solving</u> behavior. Both originate in this primitive separation". According to Piaget (1970) play is the "assimilation of reality into the self." Ginsburg and Opper (1979) suggest the concrete example of a predominance of accomodation is imitation. A child in play may "pretend" that two sticks crossed together is an airplane (he/she has assimilated the object into an airplane scheme). A child may imitate the shape of a fish hook by bending his/her finger (he/she has accomodated the finger-bending action scheme to a fish hook). In adults intellectual play may take the form of "brainstorming" where unusual connections, and diverse ideas are generated, usually by noting similarities between diverse, unrelated things. In adults imitation may take the form of "cookbook" problem solving, where one's actions accomodate directly to step-by-step instructions. Rote memorization becomes simply an imitation of previously perceived material.

McCormick (Note 2) attempted using the concepts described in the last paragraph to measure, via a self-report questionnaire, the degree to which assimilation and accomodation are generally in balance among students during the course of studying graduate level statistics. Questions were also inspired by Sample's (1968, 1976) balance of metaphorical mode (or playful) vs. substantive mode (or work) in natural problem solving behavior of children. Both Piaget's and Sample's play-like components of problem solving involve seeing (or assimilating) similarities between the external material and an internal scheme or metaphor. As mentioned before, Piaget suggests that a nonbalance occurs when "an obstacle" holds an "assimilation in check". One cannot assimilate the elements of the external reality unless one has the systems to do so. Just as the grasping action scheme accomodates to a large object by having the hand open wide, a student accomodates his/her conceptual schemes to extremely difficult subject matter through imitation. He/she may imitate the problem solving procedures given by the teacher; he/she may take prolific notes as a verbal imitation of the teacher's knowledge; he/she may read passages out of the text numerous times until he/she is capable of imitating verbatum the text. These methods in lexical terms are simply methods of "rote memorization." McCormick's (Note 2) questions pertaining to a general nonbalance in favor of accomodation stressed this memorization aspect.

A student who takes a very playful approach to the course material, like a child at play, will see similarities in just about everything. But again, just as mentioned before, Piaget (1977) suggests that the initial cause of nonbalance is "the systematic primacy of the positive characteristics" and the "assimilation at the beginning of affirmations almost exclusively." That is, a student will be unbalanced by assimilating only the similarities between the course material and the "scheme of assimilation" he/she chooses. It is on this perception of similarities between course material and other unrelated material that McCormick (Note 2) based his questions pertaining to a nonbalance in favor of assimilation.

Additional questions pertaining to nonbalance in favor of assimilation have been based on some student's use of concrete examples. Students tend to use a concrete situation in order to assimilate abstract concepts. The nonbalance persists however until the student accomodates the concrete example into a conceptual scheme generalizable to other concrete situations.

Generally, those unbalanced in favor of product (accomodative, imitative) would have difficulty applying the material that they are imitating to new situations. So, a test that includes levels of application and syntheses of knowledge would favor those balanced away from an accomodative orientation, and towards an orientation where new information is more readily assimilated, i.e. the balanced or play orientations.

<u>M-space as a Main Effect</u>

Pascual-Leone's (1970) concept of M-space evolved from an idea developed by Miller (1956). This idea is that the maximum number of "chunks" of information which can be held in short-term memory (STM) is "seven plus or minus two" for an adult. This number is referred to as M-space by Pascual-Leone. He suggests in particular that M-space is the number of figurative schemes (internalized imitations of the "external reality") plus operative schemes (transformations applied to figurative schemes to obtain another figurative scheme). An executive scheme involves strategies that coordinate operative and figurative schemes, and presumably require no space in STM. Pascual-Leone posits that the size of M-space is determined developmentally. A four year old child would have an M-space of $e+1$, where e is the number of executive schemes in use, and 1 is the number of other schemes called to STM. M-space increases by one every two years until at sixteen one's M-space is $e+7$,(a normative expectation). Learning and problem solving depends among other things on the capacity of STM, since it is presumed that it is when schemes are in STM together that they undergo reciprocal assimilation and accomodation or "chunking".

Case (1972) interprets Pascual-Leone's sufficient conditions for problem solving as:

"(1) The person must have the appropriate schemes in his /her repertoire. The construction of these schemes is interpreted by Pascual-Leone as learning.

"(2) The person must have a tendency to use his full M-space and to avoid the exclusive activation of schemes triggered by perceptually salient features of the problem. This tendency is interpreted by Pascual-Leone as field independence." (after Witkin et. al, 1962).

"(3) The person must have a tendency, when two incompatible schemes might be activated, to activate only that scheme which is compatible with the greatest number of other schemes. This tendency is assumed to be universal, and is interpreted by Pascual-Leone as an essential component of equilibration."

A determining factor in whether M-space will be shown to be a predictor in problem solving is whether a person is field independent. Field independence is the ability to deal with a perceptual configuration independent of an overlapping configuration or background field. Field independence is usually measured by "The Embedded Figures Test" (EFT) or the "Rod and Frame Test." Such tests load very heavily on a spatial abilities factor (Guay

& McDaniel, 1977; McGee, 1979). So it may be that M-space predicts problem solving and learning skills only in the presence of (1) field independent cognitive style or (2) high spatial ability. In experiments performed by Case (1974, 1977) deRibaupierre (1975), and Pascual-Leone (1970), subjects were measured for their field independence. Lawson (1976) used subjects irrespective of field independence and found M-space to be less reliable in predicting problem solving outcomes. McCormick (Note 2) found M-space to be of marginal reliability as a main effect predicting course work scores in graduate statistics, but found a strong interaction of M-space with spatial abilities.

Case (1977) used 63 "non-conserving" (unable to say whether quantity of a substance remains the same after a change of shape or containers) children aged 5 to 8 in an experiment to test the effects of subjective uncertainty and conflict exercises on learning a conservation task. Covariates in the design were M-space (Backward Digit Span), field independence (Rod and Frame Test) and impulsivity (Matching Familiar Figures Test). Case indicated that the differences on the conservation posttest between the treatment and control groups were significant ($\chi^2(1)=5.96$, $p<.02$). "When grade and age were removed..., the residual correlation of M-space with post-

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*Satterly, 1976 found EFT in a factor distinct from Spatial Abilities. However, EFT had a strong correlation with all tests he used as well as the spatial tests.

test performance was still significant (additional variance accounted for = 10%, $p < .006$)." Field independence and impulsivity had no significant effect, in this sample. It was expected that subjects require an M-space of e+3 in order to perform on a conservation task. Case said that "non-conservers with an M-space of e+3 are difficult to locate." The result of the training was that none of the children with M-space=e+1 showed "evidence of conservation." Twelve out of 41 subjects with M-space=e+2 attained conservation, while 5 out of 6 with M-space=e+3 attained conservation.

Case (1975) points out that the minimal requirement, if proper instruction is used, is an M-space equal e+2. If a concept is analyzed into component schemes, which are already in the person's repertoire of schemes, then the components can be "chunked" (i.e. undergo reciprocal assimilation and accomodation) two at a time until the entire concept is constructed. What is needed of course is someone to analyze that concept into the components in the first place. However, it seems at this point that humans may never be able to grasp complex concepts unless there is a super-human to analyze the concept into component concepts that we now grasp. But we do have heuristic devices like pencil and paper, pictures, models, etc. with which we attempt to analyze a complex problem and put it back together again piece by piece.

The test used by Case to measure M-space is called Backwards Digit Span. Numbers are read off, e.g. 5-7-2-6, and the subject is to reverse the digits in his mind quickly and respond, e.g. 6-2-7-5, in writing or orally. The reversing process is used to avoid automatic "chunking" that may occur. For example 6-8-7-3-7-1-2-0-5-6 (eleven digits) may be chunked into two numbers by the present writer (telephone and house numbers), and read back quickly had forward digit span been used. A complaint against the use of Backward Digit Span concerns the emphasis on the figurative aspects of what is to be recalled, as opposed to Pascual-Leone's theoretical requirement of both figurative and operative schemes (Lawson, 1976). Backwards Digit Span tends to be used for its convenience, and not just because it is impossible to observe directly anything other than the figurative aspects of another person's cognitive processes*.

In summary, M-space as a main effect is expected to be of little reliability in predicting outcome of graduate level statistics for the following reasons: (1) Pascual-Leone suggests that the M-space effects will be seen primarily among field independent persons. Hence, student samples which are heterogeneous with respect to field independence would result in uncertain predictability. (2) Case suggests that an appropriate teaching method can

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*Piaget (1977) would possibly suggest that our observing such figurative aspects themselves depends on our own operative schemes. Even one must infer that one controls one's own actions!

overcome the problem of a below-average M-space. If an instructor analyzes concepts into components, and serially and pair-wise brings the components together, then all the student needs is an appropriately minimal conceptual repertoire and an M-space of e+2. (3) Students who have a learning strategy that is similar to Case's analytic-serial teaching strategy do not of necessity need an M-space larger than e+2. Because of these three points the present writer expects M-space to have effects only in combination with other abilities and learning and teaching strategies to account for variance in graduate level statistics scores.

Spatial Ability as a Main Effect

An incredible amount of research has already been conducted with regard to "spatial ability". Smith (1964) and McGee (1979) both have conducted comprehensive reviews, and yet overlooked many relevant studies. McGee, for example, reviewed over 300 books and articles on spatial ability but still failed to mention Piaget and Inhelder's (1971) important work related to this area. The topic seems insurmountable. Attention, therefore, will be highly focused and relevant to the study at hand.

McGee concludes that spatial abilities has two primary factors: visualization and orientation. Spatial orientation is "the comprehension of the arrangement of elements within a visual stimulus pattern and the aptitude to remain unconfused by the changing orientation in which a spatial configuration may be presented." Spatial visualization is "the ability to mentally manipulate, rotate, twist, or invert a pictorially presented stimulus object." Smith (1964) suggests that spatial ability is required to make configural transformations in two and three dimensions. He also suggests that spatial ability is highly related to reasoning ability, and to the ability to grasp the Gestalt of a situation, or think holistically. Piaget and Inhelder (1971) suggest that a child's mental imagery is linked to his/her developmental level, and hence his/her abilities of problem solving.

There are many studies relating spatial abilities and math achievement, and several relating to college level math. Much of the work involving spatial abilities tend to be simply correlational in nature rather than experimental. Examples of this sort are found in work by Das, Kirby, and Jarman (1975); Kirby and Das (1978); Smith (1964) and Satterly (1976). Satterly's investigation is interesting for what it does not show as well as what it does show.

Satterly attempts to show that such tests as Embedded Figures Test (EFT) and Analytic Style Preference Tests measure a cognitive style factor distinct from intelligence and spatial ability. Contrary to studies and reviews, mentioned before, EFT did not load on his spatial factor, but did load on his intelligence factor. What makes the results suspicious is the factor loading of English comprehension on his spatial factor. Again contrary to numerous studies (McGee, 1979) Mathematics achievement did not load at all on his spatial factor. It is such descriptive studies as this, which use questionable measures of spatial abilities and have contradictory results, that obscure the significance of a spatial ability measure.

Das, Kirby, and Jarman (1975), Kirby and Das (1978) and Das, Kirby, and Jarman (1979) collected data, which included scores from tests that have in the past been used to measure spatial or imagery abilities. Their general theory involves processes to maintain arousal states, to plan and to encode information. Information may be encoded under the influences of two processes: simultaneous syntheses and successive syntheses. Kirby and Das (1978) "found that simultaneous processing was primarily related to spatial ability". They state: "Briefly, simultaneous processing can be characterized as involving the synthesis of separate elements into groups that generally have spatial overtones, with all portions of the synthesis being surveyable or accessible without dependence on their position within the synthesis. This type of processing is required, for instance, in the formation of any holistic gestalt, or in the discovery of the relationships among two or more objects. Successive processing, on the other hand, involves the integration of the separate elements into groups whose essential nature is temporal. Portions of this synthesis are accessible only in the temporal order of the series - each element leads to only one other, and access to any element is dependent on the preceding elements."

At this point it should be noted that a spatial test (e.g. Paper folding test; French, Ekstrom & Price, 1963) loads heavily on simultaneous synthesis. (Das, Kirby and Jarman, 1975). A test of M-space (Backwards digit span) may load heavily on the successive synthesis factor just as forward digit span and concrete word and abstract word memory span. (Das, Kirby and Jarman, 1975) Das and his collegues suggest that these syntheses are the modalities in which one processes information. This processing choice (simultaneous or successive) is decided upon in a "planning" block of the brain, and they do not suggest necessarily that some people are good at, say, simultaneous processing while being poor at successive, etc. (Das, Kirby and Jarman, 1979). But McCormick's (Note 2) data do suggest that there are people who have above average scores on Paper Folding Test while having below average scores on Backwards Digit Span. It is difficult to say whether this is above average simultaneous and below average successive processing, rather than inconsistent planning of strategies.

Piaget and Inhelder's (1971) data on the development of mental imagery in children identifies the influence of operative thought on static mental images. Before a child obtains concrete operational thought (Ginsburg & Opper, 1979) at about 7 years of age, he/she is able to produce roughly correct mental images of static configurations. It tends to be static, reproductive imagery. As the child obtains higher levels of operative thought, the static image comes under influence of the operations. By the time the child reaches formal operative thought past 12 years of age, he/she will have developed from these static, reproductive images into kinetic (moving), reproductive imagery into transformational, reproductive imagery. Overlapping with this development is a component of imagery that seems to be related to type Y behaviors described above. This is an anticipatory type of imagery. Piaget and Inhelder do not describe this as stage-like development other than the stage-like development of operative thought. However, reproducing a static image in mind occurs at the lowest level of development. Reproducing a transformational image (e.g. visualization of paper folding) is simpler than anticipating a transformational image (e.g. visualization of paper being folded,punched with a hole,and unfolded). The ability to perform on an anticipatory, transformational imagery appears to this writer very much related to the more advanced levels of type Y behaviors.

A test of spatial ability most appealing to this writer is one in which the examinee has to 1) form and hold a static, mental image and 2) anticipate transformations on that image, 3)under a pressure of time so that holistic (or simultaneous) processes need be called into play for optimum success. Paper Folding Test (French, Ekstrom & Price, 1963) and Space Relations (Differential Aptitude Tests) are two tests of spatial visualization most familiar to the present writer. Both meet the three requirements above. Because of their ease in administration and scoring, such excessive error of measurement need not be introduced.

Interactions of Variables

No published research could be found that explores the interaction of spatial abilities with M-space on any criterion. A related interaction, that of simultaneous by successive synthesis, has not been found at least in the literature. McCormick (Note 2) combined a double interaction of spatial by M-space with a triple interaction of balance by spatial ability by M-space to get a single interaction as

(1- Balance)*Mspace*Spatial

Note that when Balance is at the mean value of zero , the expression reverts to the double interaction. This combined interaction uniquely accounted for more than 20% of the variance in course score (multiple regression for the social sciences) ($F=13.63$, $df=1,24$, $p<0.002$). The combined double interaction effect is plotted (Figure 1) as three, two-dimensional manifolds of a four-dimensional hypersurface.

Pask (1976) considers directly the interaction of teaching strategy with learning strategy. In his series of experiments using programmed instruction and machine instruction he selects students according to their definite learning strategy. These strategies are holistic vs. serialistic. The "holists" or "comprehension' learners readily pick up an overall picture of the subject matter, for example, redundancies in a taxonomic scheme or relations between distinguished classes...These individuals are able to build descriptions of topics and to describe the relation between topics...Their cognitive repertoire includes effective though individually distinctive building operations, although such learners may not be able to apply these operations... until the procedures underlying the concepts in question are specifically taught."

In speaking of the "serialists" he states, "Left to their own devices, operation learners pick up rules, methods details, but are often unaware of how or why they fit together."

Pask (1976) matched or mis-matched learning style with teaching style to see the effect on outcome. The results were as expected. The matched cases far outperformed their un-matched counterparts. These data do not suggest (even though Pask pointed this out) that students are competent in the learning strategy that they chose for themselves. To be competent at a holistic strategy it would seem that one would need the ability to <u>think</u> holistically. It is here believed that it would be more interesting to compare teaching strategy (holistic vs. serialistic) and its interaction with the ability to <u>think</u> holistically (spatial ability) on a suitable criterion (e.g. graduate level statistics for the social sciences).

CHAPTER THREE

Method

The experimenter assumed the teaching function of one professor in two classes of graduate level statistics for the social sciences for twelve teaching hours. During those periods the experimenter lectured in the following topic areas: measures of frequency, graphing, measures of central tendency and measures of dispersion. The two classes received the same number of hours of instruction, over the same topic domain, except one class received instruction using the "holistic strategy" (described above) while the second class received instruction using the "serialistic strategy". The two instructional strategies were <u>operationally</u> defined by the number of aspects of a subtopic that was discussed before the next subtopic was mentioned. This number is obviously significantly fewer for the holistic strategy than for the serialistic (see Appendix A). All subtopics are discussed concurrently in the holistic strategy.

There were four criterion variables of interest. One was the total score of all questions on a statistics exam covering the above mentioned topics (hereafter called "TOTAL"); the second was the score from multiple choice questions (hereafter called "MCHOICE"); the third was the score from a computation section of the exam (hereafter called "COMPUTE"); and the fourth was the score from a problem that required the synthesis of concepts in an unusual way (hereafter called "SYNTH").

The "Balance" questionnaire (McCormick, Note 2), mentioned above, underwent a separate reliability study of its own. The final version of the Balance questionnaire was compared using factor analysis with the four factor Inventory of Learning Processes (ILP) of Schmeck, Ribich and Ramanaiak (1976).

<u>Sample</u>

Subjects were 33 students of a graduate course in Inferential Statistics at a Midwestern university. There were 20 male and 13 female students in the sample. Previous math experience ranged from zero to six semesters (mean = 1.6, S.E. = 0.4) beyond undergraduate college algebra. Range of age was from 23 to 35 years of age. The class receiving the serialistic strategy contributed 20 students to the sample, while the holistic strategy contributed 13 students. The choice for which teaching strategy a class would receive was determined by a "toss of a coin".

In order to control for inherent differences between sections various other variables were measured. Besides variables described below the following are included: (1) the average time spent in study (recorded each day and then averaged); (2) excess study before exam (ratio of time spent in study during three days before exam to that during first week of class); (3) number of semesters of college math experience beyond college algebra; (4) number of classes attended.

Instruments

Copies of the following instruments are in Appendix B:

1. Backwards Digit Span: From a tape recording examinee listens to numbers being voiced. Two seconds after the last digit is read the voice instructs "write". Examinee then writes in reverse order the numbers that were voiced. The test presumably measures the number of schemes (in this case digits) that can be held in short term memory at one time. The instruction to reverse the serial order of the digits and the short amount of time allowed, reduces "chunking" effects (Case, 1974).

2. Paper Folding Test 1 (French, Ekstrom & Price, 1963): Examinees are given successive diagrams of a piece of paper being folded and punched with a hole. Examinee is to select from a set of distractors the diagram that represents the paper unfolded with holes. The test is one test when combined with the following test yields a spatial ability score. The test is timed.

3. Space Relations (Differential Aptitude Test): Examinees are given the flat unfolded surface of a solid

object. Examinee is to select from a set of distractors those views of objects which may possibly be the original object once it is folded into its solid shape. The sum of the standardized scores of Space Relations and Paper Folding 1 forms the spatial ability score.

4. Balance Questionnaire: (Evolved from McCormick, Note 2). Questions pertaining to the degree students generally readily assimilate statistics material or have to accomodate to the material first via memorization, repetition and work attitude. Words related to affect towards statistics are kept to a minimum. Examinee responds with numbers appropriate to whether they disagree, slightly agree, agree or strongly agree to the descriptive statement.

EXAMPLE: I don't take notes in statistics
lecture because everything seems
to "just fit" into what I already
understand.

The difference between the assimilation and accomodation scores form the balance score.

Reliabilities for the assimilation and accomodation portions of the questionnaire is .75 and .85 respectively. This gives a reliability 0.88 for the questionnaire as a whole.

5. The Criteria Measures: The professor of the course (not the experimenter) wrote two equivalent forms of a

statistics exam that would cover the lecture material. The two forms were distributed at random among both classes at exam time, so that roughly half of each class received one or the other form of the test. In-so-far as raw scores, there were no differences between forms of the exam with respect to TOTAL, MCHOICE, COMPUTE, and SYNTH. Several t-tests of significance were used in testing a hypothetical difference in the two forms (all t-values $|t| \leq 0.81$, $p > 0.42$). The use of two forms in this fashion obviates results that are due solely to artifacts of the criteria measures.

Method of Analysis

Reliability of the Balance questionnaire was assessed by calculating the inter-item reliability for the "assimilation" and "accomodation" portions separately. Also, the individual questions were factor analyzed to determine the pattern of relationships among the questions.

The "assimilation" and "accomodation" portions were factor analyzed in conjunction with Schmeck, Ribich, and Ramanaiah's (1977) four factors on their Inventory of Learning Processes. These factors are Synthesis-Analysis ("organizational processes"), Elaborative Processing ("active, elaborative approaches to encoding"), Study Methods (adherence to systematic, traditional study techniques), and Fact Retention ("the propensity to retain detailed, factual information").

The four hypotheses described in Chapter One were analyzed as follows:

<u>Hypothesis I</u>: Sample from McCormick (Note 2) formed a norming group on which M-space, Spatial score and Balance score was standardized. The Balance score was based on eight questions from the former Balance Questionnaire (McCormick, Note 2). These scores were then standardized z-scores, substituted into the equation

$$\text{Statistics} = 0.4 * \text{Spatial} + 0.4 * \text{Mspace} + 0.4 * \text{Balance} + 0.5\,(1-\text{Balance}) * \text{Spatial} * \text{Mspace}$$

A correlation between this predicted statistics score and the actual statistics score will be calculated. A t-test of significance of correlation (Roscoe, 1975) will determine whether this correlation differs significantly from zero at $\alpha = .05$.

<u>Hypothesis II</u>: Hypotheses a), b), and c) will be tested via multiple linear regression using the following full model:

$$\text{Stats} = \alpha + \beta_1 \text{Spatial} + \beta_2 * \text{Mspace} + \beta_3 * \text{Balance} \quad \beta_4 * T + \beta_5 T * \text{Spatial} + \beta_6 * \text{Spatial} * \text{Mspace} + \beta_7 * \text{Spatial} * \text{Mspace} * \text{Balance} + \beta_8 * \text{Balance} * \text{Mspace} * \text{Spatial} * T + \varepsilon_1$$

where Stats = test score results TOTAL, MCHOICE, COMPUTE and SYNTH.

Spatial = spatial ability score

Mspace = M-space

Balance = discrepancy between general assimilation of statistics and accomodation to statistics

T = treatment contrast between holistic & serialistic teaching strategies

<u>Hypothesis IIa)</u>: Ho: $\beta_5 = 0$, A: $\beta_5 \neq 0$

Restricted Model 1 (T*Spatial removed):

$$Stats = \alpha + \beta_1 * Spatial + \beta_2 * Mspace + \beta_3 * Balance + \beta_4 * T + \beta_6 * Spatial * Mspace + \beta_7 * Mspace * Spatial * Balance + \beta_8 * Balance * Mspace * Spatial * T + \varepsilon_2$$

The test of significance will be

$$F = \frac{(E_2SS - E_1SS)}{E_1SS / (N-9)}, \quad df = 1, (N-9), p < .05$$

where N = number of subjects

E_2SS = error sum of squares restricted model 1

E_1SS = error sum of squares full model

<u>Hypothesis IIb)</u>: Ho: $\beta_7 = 0$, A: $\beta_7 \neq 0$

Restricted Model 2 (Mspace*Spatial*Balance removed):

$$Stats = \alpha + \beta_1 * Spatial + \beta_2 * Mspace + \beta_3 * Balance + \beta_4 * T + \beta_6 * Spatial * Mspace + \beta_4 * Spatial * T + \beta_8 * Balance * Mspace * Spatial * T + \varepsilon_5$$

The test of significance will be

$$F = \frac{(E_3SS - E_1SS)}{E_1SS / (N-9)}, \quad df = 1, (N-9), p < .05$$

where E_3SS = error sum of squares restricted model 2.

<u>Hypothesis IIc)</u>: Ho: $\beta_8 = 0$, A: $\beta_8 \neq 0$

Restricted Model 3 (Balance*Mspace*Spatial*T removed):

$$Stats = \alpha + \beta_1 * Spatial + \beta_2 * Mspace + \beta_3 * Balance$$
$$+ \beta_4 * T + \beta_6 * Spatial * Mspace + \beta_5 * Spatial * T$$
$$+ \beta_7 * Mspace * Spatial * Balance + e_4$$

The test of signficance will be

$$F = \frac{(E_4SS - E_1SS)}{E_1SS / (N-9)} , \quad df = 1, (N-9), p < .05$$

where E_4SS = error sum of squares restricted model 3.

CHAPTER FOUR

Results

Balance Reliability Study

The Balance Questionnaire underwent three reliability trials to arrive at its final form. The following is a summary of the reliabilities obtained in each trial.

Trial	#Items	N	Assimilation	Accomodation	Balance
1	10	31	0.75	0.85	0.88
2	14	20	0.70	0.77	0.85
3	14	33	0.56	0.73	0.78

The apparent drop in the "Assimilation" reliability from 0.70 to 0.56 is not significant (Fisher's Z, $t=0.77$, $p>0.44$), and may be accounted for by sample variability.

A factor analysis was performed using the subscale scores from Balance (Assimilation and Accomodation) and from Schmeck, Ribich and Ramanaiah's (1976) Inventory of Learning Processes (Synthesis-Analysis, Study Methods, Elaborative Processes, and Fact Retention). An iterative principal axis method with varimax rotation was performed, and three factors were retained. As can be seen in Table 1, of the three factors retained, none had either "Accomodation" or Fact Retention loading greatly on it. Factor 1 appeared to be well defined by Synthesis-Analysis and Elaborative Processes. Factor 2 appeared to be well defined by "Assimilation", though there appeared trace variances of Fact Retention and "Accomodation." However the small communalities of Fact Retention and "Accomodation" obviate against suggesting anything stronger. Factor 3 was well defined by Study Methods with traces of Elaborative Processes and "Assimilation". The correlations (Table 2) among these six subscales are all non-significant. Consequently, it appears that there are no solid data to suggest that the subscales of Balance are related to one or several of the subscales of the ILP.

Differences Between Classes

Table 3 lists all variables that were measured, their means and standard deviations for both class sections combined and for the two sections separately. Also, t-tests were performed to ascertain uncontrolled differences between sections on all these variables. While none of the measured variables were significantly different between sections, several variables were different relative to variability. F-tests on all variables between sections were performed. Section 1 appeared to be more variable in the number of classes attended than Section 2. Also, Section 2 appeared more variable than Section 1 with respect to excess study before the exam. However, this may be accounted for by one student who did not study until the day before the exam, when he studied for 14 hours.

Cross-validation: Hypothesis I

The regression equation derived by McCormick

(Note 2),

$$Stats = 0.4 * SPATIAL + 0.4 * MSPACE + 0.4 * BALANCE + 0.5 * (1-BALANCE) * MSPACE * BALANCE,$$

was used in an attempt to predict exam outcome for the present group. With the present group there were four criteria measures under consideration. The correlation between predicted statistics grade and the total score was significant ($r = 0.505$, $p<0.003$). However, this is only a 3% improvement in variance accounted for over a single predictor model using the Spatial score alone (Table 2). The correlation between the multiple choice score and the predicted score was significant ($r = 0.467$, $p<0.007$), but not very different from the correlation between multiple choice and spatial score (Table 2). The computation score was as well significantly correlated with the predicted score ($r = 0.455$, $p<0.008$). In this case the prediction model accounted for about 15% more variance than had the spatial score alone been used. The correlation between the predicted statistics score and the question involving the synthesis of concepts in a unusual situation did not significantly differ from a zero value ($r = 0.272$, $p<0.12$). In fact the only variable with which this synthesis question correlated significantly was the ILP: <u>Study Methods</u>, $r = -0.356$, $p<0.05$ (Table 2).

Complex Interactions: Hypotheses IIa, b and c

Statistical models were constructed via multiple regression as per Hypothesis II using TOTAL (the total exam score of the material covered), MCHOICE (the score on the multiple choice questions), COMPUTE (the score on the computation questions) and SYNTH (the score on the synthesis questions), all described above, as criteria measures.

Table 5 is the analysis of variance of the hypothesized model using MCHOICE as the criterion of the three effects of interest. Only the quadruple interaction of class section by M-space by Spatial by Balance rose to level of signficance ($F=5.90$, $df=(1,24)$, $p<0.015$). This effect accounted for 11.4% of the variance. Tests of signficance for lower order interactions and main effects which are components of this interaction have no meaning in the presence of this interaction, other than being subsumed by it. The full model R -square was 0.535 ($F=3.46$ ($df=8,24$), $p< 0.009$). This regression model is stated with Figure 2, which is a graphical plotting of this equation as four two-dimensional manifolds (i.e. a section or "slice")of a five-dimensional hyper-surface. The hyper-surface is a least squares fit of the data points set in the five-dimensional space. The reader is cautioned against attempting to interpret the graphs of Figure 2 (and Figures 3 and 4) as lines of best fit through groups of given characteristics. It may be noted that at the high end of the Balance dimension (Balance=+2σ=8.84 raw value) the hypersurface tends to warp in the clockwise direction when going from the serialistic towards the holistic teaching strategy. While the warping is in the opposite sense at the low end of the Balance dimension. It appears, therefore, that spatial ability is a determining factor for students with an assimilative attitude under the serialistic strategy and for students with an accomodative attitude in the holistic strategy. Also, it can be noted that for MCHOICE, students with higher M-space tended to do generally better than lower M-space students. There are several regions, however, in which differences because of M-space are not apparently noticable.

Table 6 is the analysis of variance of the hypothesized model using COMPUTE as the criterion. The quadruple interaction uncovered for MCHOICE rose to signficance but in smaller degree ($F=3.68$, $df=(1,24)$, $p<0.04$). Rather than make a Type II error, which is possible with small samples, and since this interaction was expected, the probability of a Type I error of (for a one-tail test), 0.05 was acceptable. The quadruple interaction accounts for 8.7% of the variance in a model that is itself leaning towards traditional signficance. ($R^2=0.433$, $F=2.29$, $df=(8,24)$, $p<0.06$). The regression equation is stated with Figure 3, which is again a graphical plot of this equation as four two-dimensional manifolds. The tendency for spatial abilities predicting COMPUTE is similar to that of MCHOICE described above. There is an interesting reversal of the effects of M-space. Because of an apparent negative weight of M-space, those with a lower M-space appear to do generally better than those with a higher M-space. This will be discussed more in-depth below.

Table 7 is the analysis of variance of the hypothesized model with SYNTH as criteria. None of the effects of interest rose to levels of acceptable significance. The Spatial by M-space interaction surfaced marginally ($F=3.09$, $df=(1,24)$, $p<0.05$), and is one of the interactions found by McCormick (Note 2). Since the model was not signficant (R =0.280, $F=1.16$, $df=(8,24)$, $p>0.35$), no graphs were made.

Table 8 is the analysis of variance with TOTAL as the criterion. TOTAL equals of the sum of MCHOICE, COMPUTE, and SYNTH. Again the quadruple interaction rose to signficance ($F=3.67$, $df=(1,24)$, $p<0.05$). Also the variable balance accounted for about 6.9% of the variance ($F=3.22$, $df=(1,24)$, $p<0.05$) in a model which did attain traditional signficance (R =0.485, $F=2.82$, $df=8,24$, $p<0.03$). This model did have a high degree of common variance (1302.206 sums of squares or about 33.3% of the total sums of squares), which means that the independent variables in the model do have a high degree of overlap among themselves. The regression equation for this model is stated on Figure 4, which is a graphical plotting of the equation as four two-dimensional manifolds. Scattergrams of observed values are plotted overlaying the graphs (keep in mind how these points are scattered through a five-dimensional space). Comparing the graphs of TOTAL to both those of COMPUTE and MCHOICE, it appears that TOTAL is a fair compromise of both COMPUTE and MCHOICE. The differences arise in the effects of M-space on the TOTAL score. Higher TOTAL scores for high M-space occurs for high-Spatial, high-Balance, serialistic teaching strategy and for high-Spatial, low-Balance, holistic teaching strategy and for low-Spatial, low-Balance, serialistic teaching strategy, and for low-Spatial, high-Balance, holistic teaching strategy (Figure 4).

Comparison to Previous Study

A comparison was made between the present sample and the sample of McCormick (Note 2). The following are the means and standard deviations of the Balance scores for the 10 questions from the two questionnaires that were in common:

	McCormick (Note 1)		Present Group	
	Mean	Stan.Deviation	Mean	Stan. Deviation
Assimilation	10.03	2.76	3.30	2.08
Accomodation	10.49	2.74	6.70	3.38
Balance	-0.46	3.89	-3.40	3.97

The difference in the Balance score between the first group and the present group is signficant ($t=2.99$, $df=62$, $p<0.01$). The present group is most likely unbalanced more toward a work or accommodative orientation to statistics. This means that there is a major change in "reference frames" required, in-so-far as Balance is concerned. The present sample was taken from the first 4 weeks of the first course of a two semester sequence. The first sample was taken from the last 4 weeks of the second course in the sequence. Attrition apparently may have selected out many of those who felt that statistics is "work" and left quite a few of those who could "play" with statistics. The transformation of reference frames then is required if any comparisons are to be made. A plus one standard deviation ($+1\sigma$) for the first group corresponds in Balance score to a plus two standard deviations ($+2\sigma$) in the present group. Similarly, a negative three standard deviations (-3σ) in the first group corresponds to negative two standard deviations in the present group (-2σ). Using the equation stated in McCormick (Note 2),

Stats=0.4 * SPATIAL + 0.4 * MSPACE = 0.4
* BALANCE + .5 * (1-BALANCE) * MSPACE
* BALANCE,

one can calculate and then graph the regression surfaces. If this is done, then predicted graphs would look like Figure 5. What has happened for the low-Balance graph (Figure 1a) is that decreasing Balance and increasing M-space tends to warp the hypersurface in a counterclockwise direction. On the other hand decreasing Balance and decreasing M-space tends to warp the hypersurface in the clockwise direction. Consequently, the M-space = 6 section of Figure 1a has moved from the 1 o'clock position to the 11 o'clock position (M-space = 8) of Figure 5a, while the M-space =4 section has moved from the 4:30 o'clock position in Figure 1a to the 7 o'clock position (M-space = 3) of Figure 5a. If an assumption (with no data to substantiate it) is made that McCormick's (Note 1) sample was taught in a serialistic fashion and tested primarily on computation, then the graphs of Figure 5 could very well be related to Figure 3 Serialistic Teaching strategy.

CHAPTER FIVE

Discussion

It is clear that the data suggest what is commonly believed, that learning is an extremely complex process. Concurrence is generally found between the results of Chapter 4 and the expectations of Chapter 1. The hypothesized quadruple interaction of teaching strategy by Spatial by M-space by Balance was found. It was quite apparent with multiple choice questions (MCHOICE) as the criterion, and to a lesser degree with computation questions (COMPUTE). None of the hypothesized interactions (in the hypothetical statistical model) were found for the synthesis of concepts question (SYNTH).

It was suggested in Chapter 1 that persons unbalanced in favor of process (assimilation or play and Balance positive) under a holistic teaching strategy would have scores that are positively related to the spatial scores irrespective of M-space. This generally is true for MCHOICE with the exception that the slope is small. The splitting due to M-space accounts for a change of about 3 points. However for COMPUTE, there is no slope, and there is a splitting for high-spatial.

Chapter 1 suggests that persons unbalanced in favor of work (accomodation, imitation and Balance negative) under a holistic strategy would do better with high-Spatial and high-M-space than low-M-space.

However, those with low-Spatial high-M-space were expected to do more poorly than low-Spatial low-M-space in this case. This is generally true for MCHOICE. But this is not the case for COMPUTE, which does have though a slight positive slope with the Spatial score.

There was a suggestion from Chapter 1 that as the balance "tilts" in favor of process (assimilation or play and Balance positive) under a serialistic teaching strategy there should be a positive correlation between statistics scores and spatial score, because "the holistically capable student may make the interrelationships among subtopics on his/her own for their own sake". As well, no effect due to M-space was expected. This was generally true for COMPUTE, but not exactly true for MCHOICE. For MCHOICE there is a small splitting effect due to M-space, with high-Spatial high-M-space doing better than high-Spatial, low-M-space.

The last suggestion from Chapter 1 was that as the balance tilts in favor of work orientation (Balance negative) under a serialistic strategy there should be a negative correlation between statistics and spatial score irrespective of M-space. This is more true of COMPUTE than MCHOICE. There appears to be a negative slope of COMPUTE on Spatial, but primarily for high-M-space.

The data for the four suggestions stated above were primarily derived from McCormick's(Note 2) sample.

It may be concluded that the instructor for that sample taught serialistically for computational material, and holistically for the factual type of material covered by MCHOICE. This conclusion is based on the observation that MCHOICE generally agrees with the holistic suggestions, while COMPUTE generally agrees with the serialistic suggestions above.

In conclusion, the study met with several successes. The cross-validation research hypothesis (in which McCormick's (Note 1) prediction equation attempted to predict exam scores) was generally accepted. The equation met with greatest success in predicting computation scores, and least success in predicting the synthesis of concepts scores (Table 4). The research hypotheses of complex interactions in learning also was generally accepted. Complex interactions were found among learning required to answer multiple choice and computation questions. However, none (beyond an M-space*Spatial interaction) were found within learning required to answer a synthesis of concepts question.

<u>Implications for Further Research</u>

While it is recognized that important policy decisions in education have been made on far less data than has been presented here, the present writer will leave to the reader to formulate implications of this research for education. It is this writer's opinion that the state of the art of complex interactions in learning does not allow one to make profound policy decisions. It will take much more careful work before a practical theory can be postulated.

The next step that should follow this study is to trace students through a sequence of statistics courses. An important factor to consider is how the characteristics of the students, who make it through the sequence, change. It was already noted that a major difference between McCormick's (Note 2) sample and the present is the apparent shift from a work orientation of the present sample towards a play orientation in McCormick's (Note 2) sample. A proper question is whether this shift is only apparent or indeed real. The strength of the quadruple interaction, that was found using the hypothesized model, alone accounted for roughly 10% of the variance. A more complete model may in the future be developed which includes contributions from other interactions as yet unmeasured. As a consequence the model will become more deterministic and hence more prophetic.

Notes

1. McCormick, R.V. and Mouw, J.T. The interaction between stimulus modality and cognitive abilitities in the solution rate of verbally stated problems. (In preparation)

2. McCormick, R.V. The balance effect on the m-space by spatial interaction. (In preparation)

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TABLE 1

FACTOR ANALYTIC COMPARISON

OF BALANCE WITH I.L.P.

ROTATED FACTOR PATTERN N=33

	FACTOR 1	FACTOR 2	FACTOR 3	COMMUNALITY
ILP: SYNTH-ANAL:	0.73320*	-0.08438	0.01385	0.54
ILP: STUDY MTHD:	0.26310	-0.07204	0.89570*	0.88
ILP: FACT RETEN:	0.00441	0.38071	-0.02415	0.15
ILP: ELABORATION:	0.81431*	0.17237	0.32071	0.80
ASSIMILATION:	-0.22126	0.81977*	0.41379	0.89
ACCOMODATION:	-0.10891	-0.35762	0.14803	0.16

TABLE 2

CORRELATION COEFFICIENTS

AMONG ALL RECORDED VARIABLES

(Decimal omitted. Underscored $p < 0.05$)

N=33

	1 Total	2 Mult. Choice	3. Computation	4. Synthesis	5. Form of Test	6. # Classes attended	7. Sex	8. # Semesters Math Exp.	9. Mspace	10. Spatial	11. ILP: Syn-Anal.	12. ILP: Study	13. ILP: Fact Ret.	14. ILP: Elabo	15 Assim.	16 Accom	17 Balance	18. Ave. Study Time
2.	908																	
3.	810	658																
4.	650	354	347															
5.	085	096	-028	111														
6.	211	186	249	075	101													
7.	150	205	022	069	-038	142												
8.	491	491	470	173	-129	144	060											
9.	198	262	196	-048	-017	-098	055	263										
10.	466	489	237	315	260	-048	204	277	153									
11.	-126	-057	-253	-043	025	059	-032	-058	-184	078								
12.	-289	-207	-158	-356	-291	287	025	146	-089	-168	237							
13.	291	394	170	022	-088	150	123	218	177	283	-009	-114						
14.	-157	-042	-076	-326	046	179	288	-093	-223	066	602	484	122					
15.	132	174	269	-155	232	294	-032	-034	410	019	-246	187	053	-019				
16.	-422	343	-509	-193	-070	001	-092	-263	-376	-407	-135	049	-167	-226	-195			
17.	396	353	530	077	167	141	057	190	493	329	-014	052	156	-042	638	-880		
18.	054	114	069	-103	040	711	-095	-080	089	-207	061	287	200	066	269	081	067	
19.	-007	-182	-007	303	-145	118	208	-174	-226	-119	189	-140	167	168	-178	-069	032	051

(19. Excess Study Time)

MEANS
BY CLASS SECTION

TABLE 3

	Both Classes N=33		Section 1 N=20		Section 2 N=13		t-Test df=31		F-Test df=12,19		Range	
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Value	p=	Value	p=	MIN	MAX
Total	63.33	11.05	64.65	12.24	61.31	15.29	0.80	0.42	1.48	0.42	34	80
Computation	21.09	3.40	21.00	3.52	21.23	3.34	-0.19	0.85	1.11	0.87	9	24
Mult. Choice	34.70	6.48	36.00	5.66	32.69	7.36	1.46	0.15	1.69	0.29	16	44
Synthesis	7.54	3.71	7.65	2.96	7.39	4.77	0.20	0.84	2.60	0.06	0	15
#of class att.	6.52	0.76	6.35	0.88	6.77	0.44	-1.82	0.07	3.98	0.02	5	7
#Semesters Math	1.52	1.79	1.60	1.98	1.38	1.50	0.33	0.74	1.74	0.32	0	6
M-Space	5.79	1.43	5.65	1.39	6.00	1.53	-0.68	0.50	1.21	0.68	3	8
Space Relations	10.12	5.45	10.25	5.02	9.92	6.26	0.16	0.86	1.55	0.37	-3	24
Paper Folding	21.39	9.48	21.25	8.86	21.62	10.73	-0.10	0.92	1.47	0.44	3	40
ILP:SYN-ANAL	14.58	2.97	14.90	2.75	14.08	3.32	0.77	0.44	1.46	0.44	8	18
ILP:Study	14.30	4.89	13.70	5.08	15.23	4.62	-0.88	0.38	1.21	0.75	2	22
ILP:Fact Ret.	5.55	1.58	5.35	1.73	5.85	1.34	-0.88	0.39	1.65	0.37	2	7
ILP:Elaboration	10.00	2.49	9.70	2.70	10.46	2.14	-0.86	0.39	1.58	0.42	2	13
Balance:Assim.	6.76	3.11	6.20	3.25	7.62	2.78	-1.29	0.20	1.37	0.58	0	15
Balance:Accom.	9.48	4.30	9.30	4.04	9.77	4.83	-0.30	0.76	1.43	0.46	1	18
Balance	-2.73	5.78	-3.10	5.75	-2.15	6.03	-0.45	0.65	1.10	0.82	-18	7
Ave Time Study	2.13	0.98	2.39	1.02	1.72	0.79	2.00	0.06	1.67	0.36	.5	5.43
Excess Study Before Exam	1.91	2.30	1.52	0.70	2.51	3.57	-0.99	0.34	25.80	0.0001	.5	14.0
Spatial	0.00	1.75	0.008	1.54	-0.01	2.10	0.03	0.97	1.86	0.21	-3.19	4.51

TABLE 4

CORRELATION BETWEEN PREDICTED

STATISTICS GRADES AND THE FOUR CRITERIA

MEASURES

(Correlation/t-test probability-Fisher's Z)

	STATS	TOTAL	MCHOICE	COMPUTE	SYNTH
STATS	1.00000 0.0000	0.50519 0.0027	0.46708 0.0061	0.45519 0.0078	0.27173 0.1261
TOTAL	0.50519 0.0027	1.00000 0.0000	0.90789 0.0001	0.81051 0.0001	0.65009 0.0001
MCHOICE	0.46708 0.0061	0.90789 0.0001	1.00000 0.0000	0.65839 0.0001	0. 3541 0.0432
COMPUTE	0.45519 0.0078	0.81051 0.0001	0.65839 0.0001	1.00000 0.0000	0.34747 0.0476
SYNTH	0.27173 0.1261	0.65009 0.0001	0.35410 0.0432	0.34747 0.0476	1.00000 0.0000

TABLE 5

HYPOTHESIZED MODEL

Criterion: Multiple Choice Questions

Model $R^2 = 0.535$, $F(8,24) = 3.46$ $p < 0.009$

Source	Independent SS	MS	DF	df= F(1,24)
Section	21.129	21.129	1	0.81
Spatial	26.353	26.353	1	1.01
Mspace	18.870	18.870	1	0.72
Balance	31.092	31.092	1	1.19
Section*Spatial	1.209	1.209	1	0.05+
Spatial*Mspace	3.916	3.916	1	0.15
Spatial*Mspace *Balance	7.298	7.298	1	0.28+
Section*Mspace *Spatial*Balance	153.632	153.632	1	5.90+*
Common	456.720	-		
Error	624.751	26.031	24	
Total	1344.970			

+ effect of interest

* $p < 0.015$

TABLE 6

HYPOTHESIZED MODEL

Criterion: Computation Questions

Model R^2 = 0.433, $F(8,24)$ = 2.29, $p<0.06$

Source	Independent SS	MS	DF	F(df=1,24)
Section	2.832	2.832	1	0.32
Spatial	4.555	4.555	1	0.52
M-space	4.124	4.124	1	0.47
Balance	69.395	69.395	1	7.92
Section*Spatial	3.599	3.599	1	0.41+
Spatial*Mspace	2.413	2.413	1	0.28
Spatial*Mspace *Balance	7.365	7.365	1	0.84+
Section*Mspace *Spatial*Balance	32.216	32.216	1	3.68+*
Common	33.906			
Error	210.322	8.763	24	
Total	370.727			

+ effect of interest

* $p<0.04$

TABLE 7

HYPOTHESIZED MODEL

Criterion: Synthesis Question

Model $R^2 = 0.280$, $F(df=8,24) = 1.16, p>0.35$

Source	Independent SS	MS	DF	F(df=1,24)
Section	2.100	2.100	1	0.16
Spatial	31.847	31.847	1	2.41
M-space	0.267	0.267	1	0.02
Balance	6.358	6.358	1	0.48
Section*Spatial	11.705	11.705	1	0.89+
Spatial*Mspace	40.847	40.847	1	3.09*
Spatial*Mspace *Balance	20.708	20.708	1	1.57+
Section*Mspace *Spatial*Balance	0.280	0.280	1	0.02+
Common	8.962			
Error	317.108	13.213	24	
Total	440.182			

+ effect of interest

* $p<0.05$

TABLE 8

HYPOTHESIZED MODEL

Criterion: Total

Model $R^2 = 0.485$, $F(df=8,24) = 2.82, p<0.03$

Source	Independent SS	MS	DF	F(df=1,24)
Section	2.144	2.144	1	0.03
Spatial	2.639	2.639	1	0.03
M-space	3.224	3.224	1	0.04
Balance	269.875	269.875	1	3.22*
Section*Spatial	0.180	0.180	1	0.00+
Spatial*M-space	8.173	8.173	1	0.10
Spatial*M-space *Balance	0.747	0.747	1	0.01+
Section*Spatial *M-space*Balance	307.697	307.697	1	3.67+*
Common	1302.206			
Error	2014.448	83.935	24	
Total	3911.333			

+ effects of interest

* p<0.05

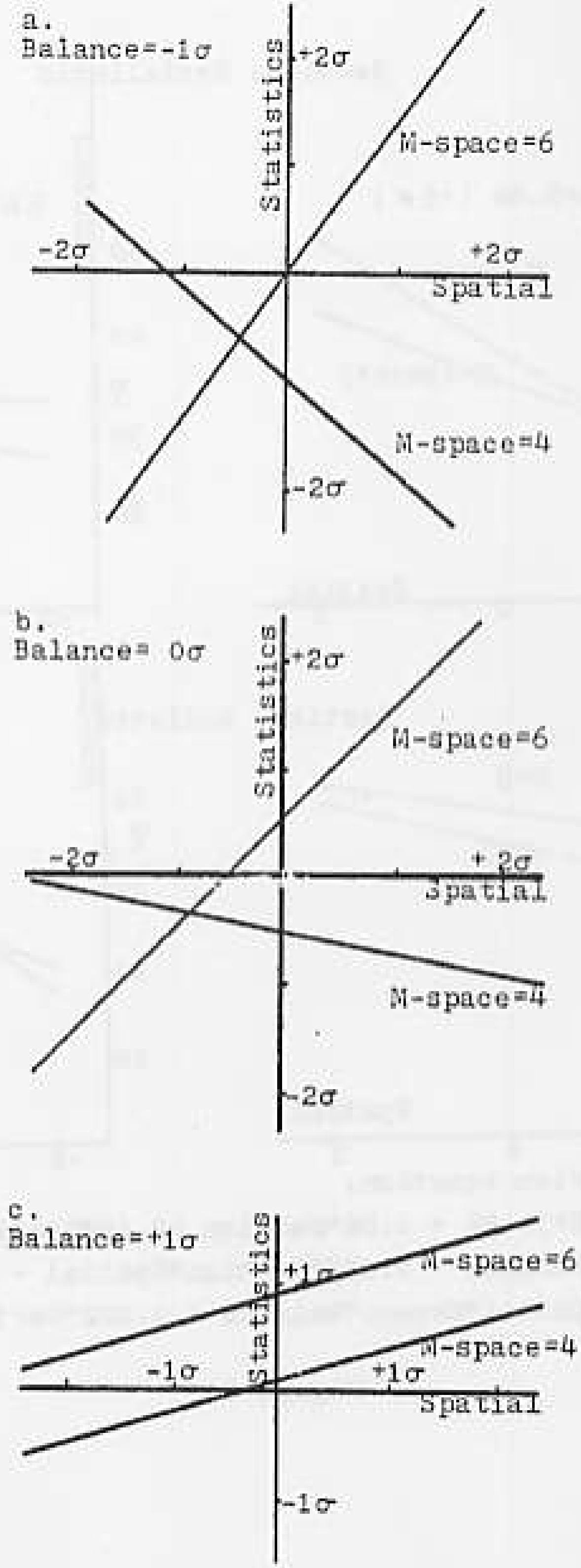
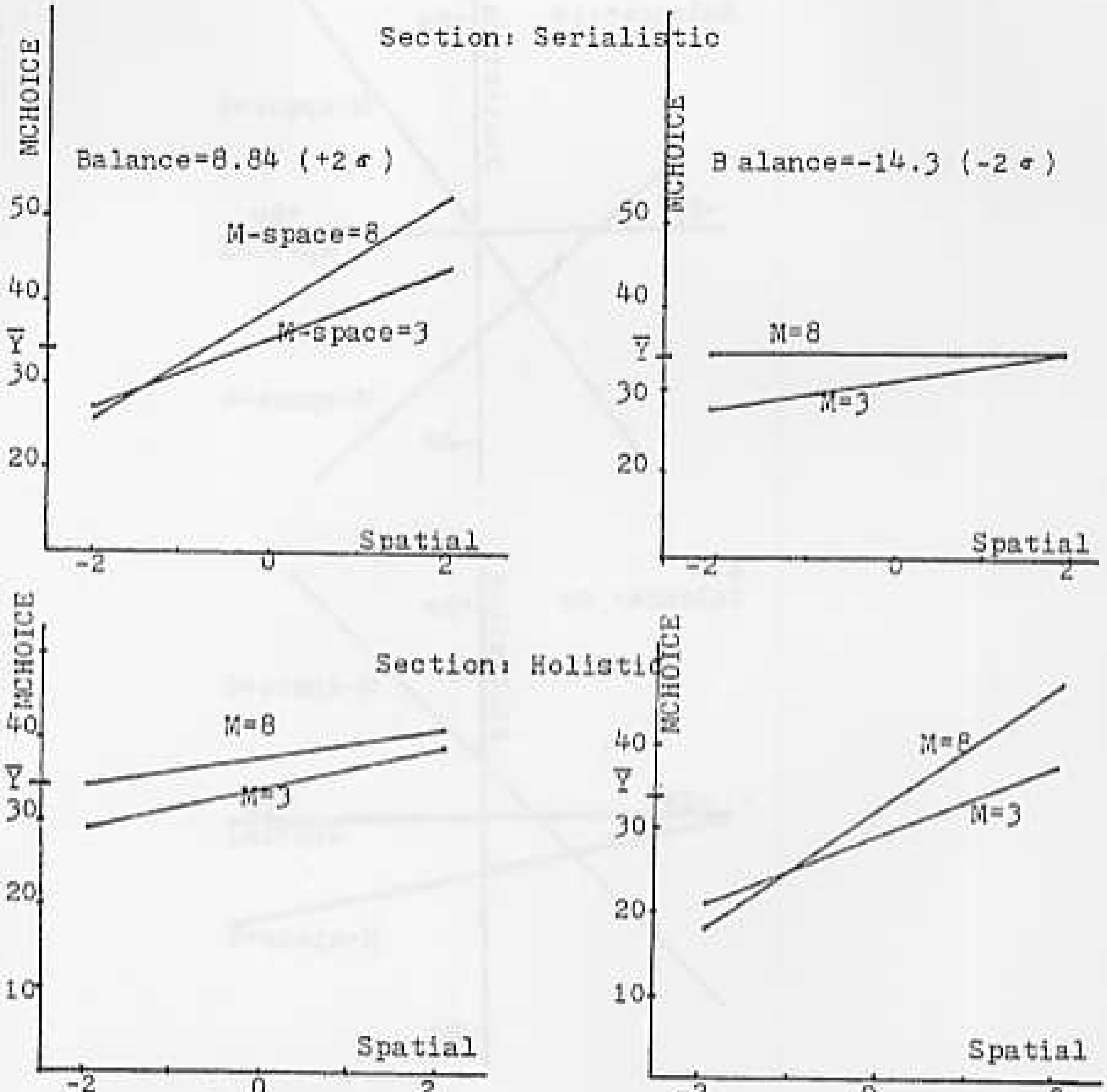

a.
Balance=-1σ
Statistics
+2σ
M-space=6
-2σ
+2σ
Spatial
M-space=4
-2σ
b.
Balance= 0σ
Statistics
+2σ
M-space=6
-2σ
+ 2σ
Spatial
M-space=4
-2σ
c.
Balance=+1σ
Statistics
+1σ
M-space=6
M-space=4
-1σ
+1σ
Spatial
-1σ

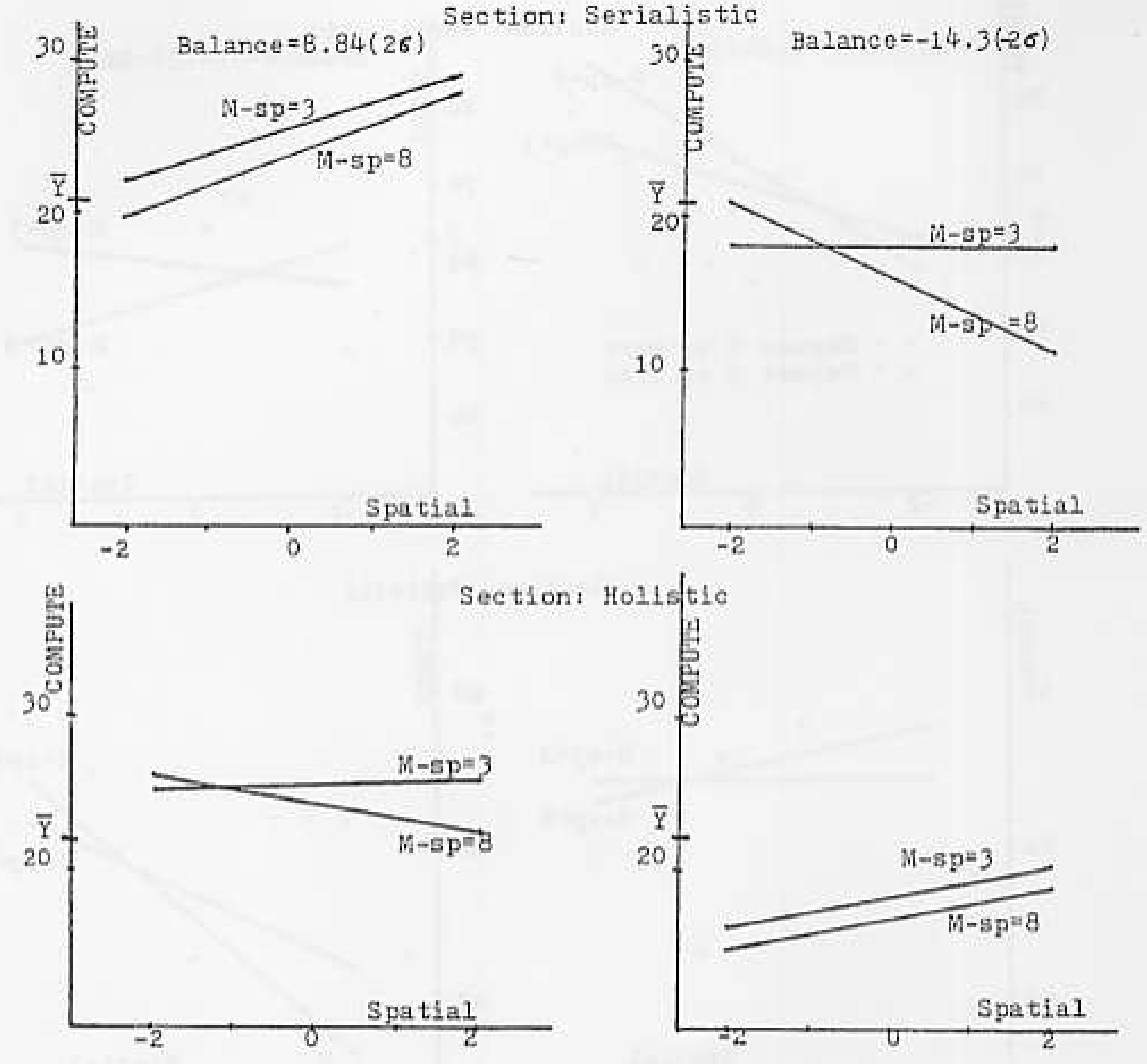
Figure 1.



Regression equation:

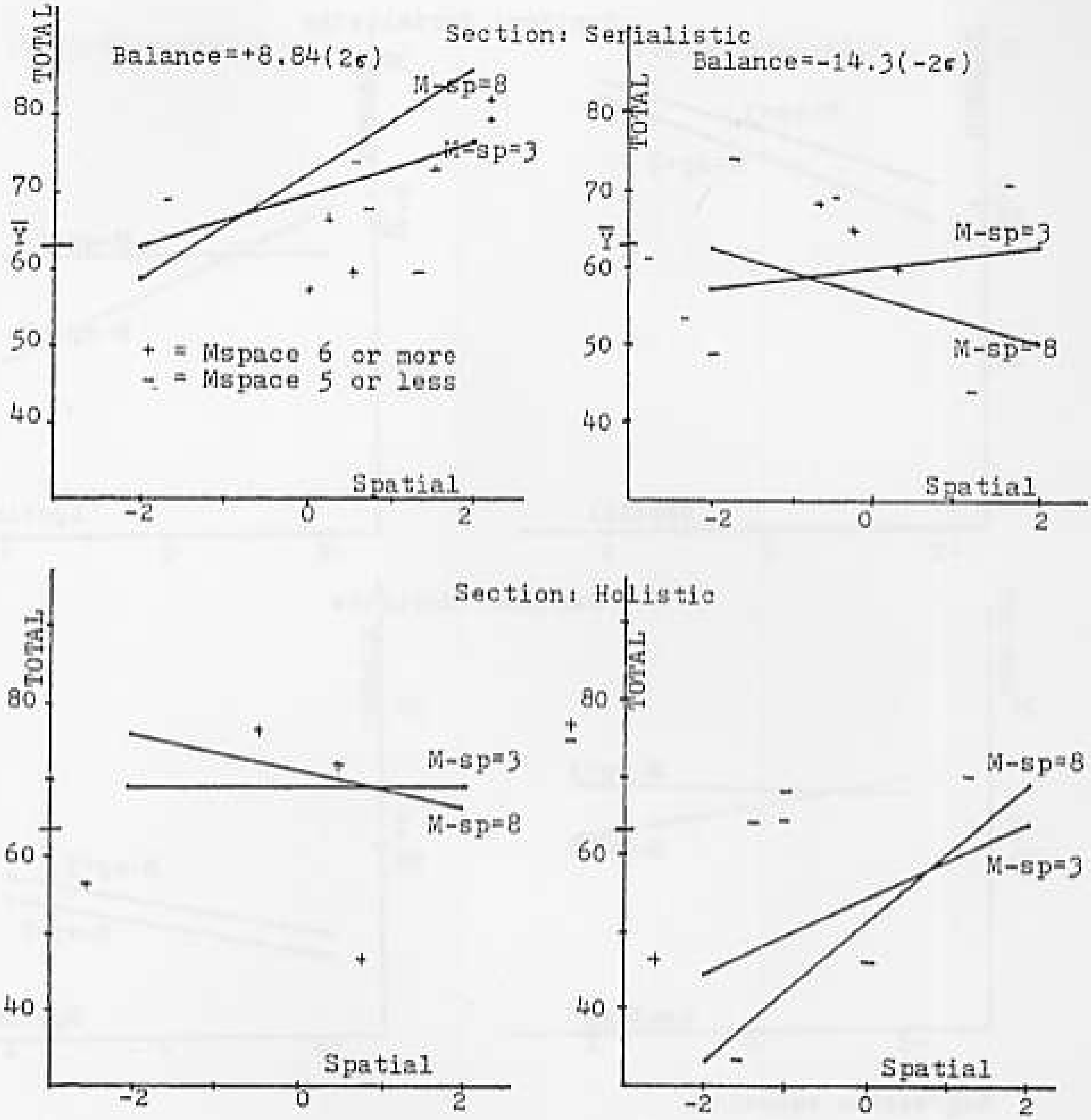
MCHOICE=31.46 + 0.06*Section +2.69*Spatial +0.69*Mspace +0.224*Balance + 0.008*Section*Spatial - 0.17*Mspace*Spatial +0.01*Spatial*Mspace*Balance + 0.002*Section*Spatial*Msp*Bal.

Figure 2.



Regression equation:
COMPUTE = 20.82 - 0.26*Section+0.17*Spatial-0.32*Mspace+0.33*Bal
+0.014*Section*Spatial-0.13*Spatial*Mspace +0.01*Spatial*Mspace*
Balance +0.001*Section*Spatial*Balance*Mspace.

Figure 3.



Regression equation:

TOTAL = 63.77 +0.02*Section +0.85*Spatial + 0.29*Mspace +0.65*
Balance -0.003*Section*Spatial + 0.25*Spatial*Mspace +
0.004*Spatial*Mspace*Balance+ 0.004*Section*Spatial*Mspace*Balance.

Figure 4.

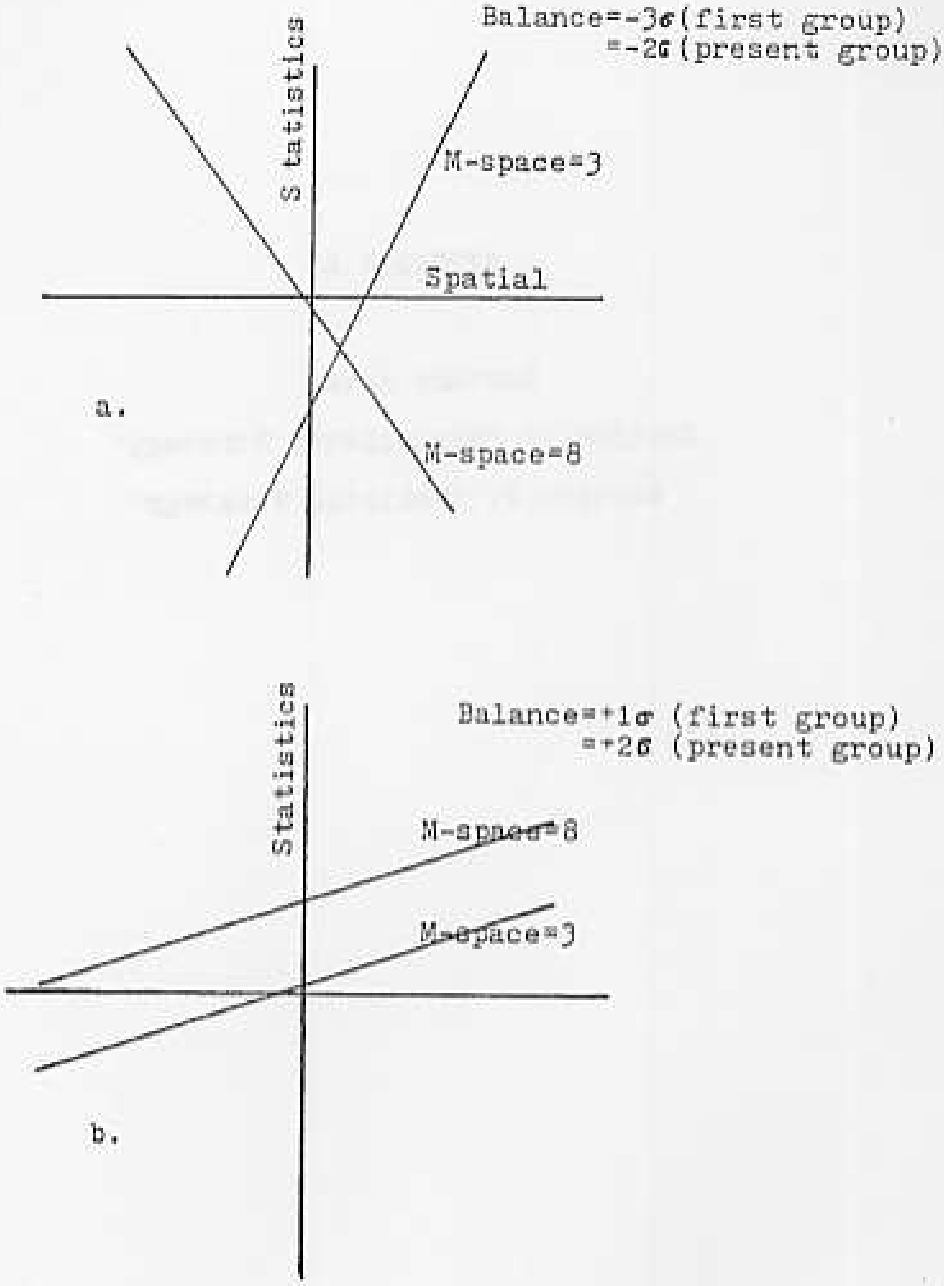


Figure 5.
Surfaces predicted by McCormick's (note 2) regression equation using present sample data.

APPENDIX A

Lecture Notes

Section 1: "Serialistic Strategy"

Section 2: "Holistic Strategy"